SIMULTANEOUS DETERMINATION OF THE THERMAL CONDUCTIVITY AND DIFFUSIVITY OF METALS USING AN ELECTRONIC HEATER

R. E. Krzhizhanovskii and N. P. Sidorova

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A nonstationary method allowing simultaneous determination of the thermal conductivity and diffusivity of metals using an electronic heater is described.

Because of the need to investigate thermophysical properties over a wide temperature range, it is very important to develop nonstationary methods which allow determination in a single experiment of the temperature dependence of one thermophysical property or another. Naturally, methods which afford the maximum information about the properties of the specimen in a single experiment are the most attractive. Another real consideration is the need to devise methods that use a convenient means of heating the specimen and generating a constant heat flux through it, i.e., an electronic heater.

These considerations have led to a new method of simultaneous determination of thermal conductivity and diffusivity of metals, based on the solution of the unsteady heat conduction problem with boundary conditions of the second kind for a semi-infinite rod.

A semi-infinite rod with uniform temperature distribution at time zero is heated by a constant heat flux of value q_e = const. The temperature varies in one direction. The equation of heat propagation in this case has the form

$$\frac{\partial t(x,\ \tau)}{\partial \tau} = a\,\frac{\partial^2 (x,\ \tau)}{\partial x^2} \quad (\tau > 0,\ 0 \leqslant x \leqslant \infty), \tag{1}$$

$$t(x,\ 0) = t_0 = \text{const}, \tag{2}$$

$$\lambda\,\frac{\partial t(0,\ \tau)}{\partial x} + q_e = 0, \tag{3}$$

$$t(\infty,\ \tau) = t_0, \tag{4}$$

$$\frac{\partial t(\infty,\ \tau)}{\partial x} = 0. \tag{5}$$

The temperature distribution along the rod has the form [1]

$$t(x,\ \tau) - t_0 = \frac{2q_e}{\lambda}\sqrt{a\tau}\ \text{ierfc}\,\frac{x}{2\sqrt{a\tau}}. \tag{6}$$

Solution (6) is valid for the case when there are no energy losses from the side surfaces of the specimen. When there are such losses, we have, instead of (1),

$$\frac{\partial t}{\partial \tau} + a\,\frac{\partial^2 t}{\partial x^2} + ct = 0, \tag{7}$$

and boundary condition (3) must be replaced by

$$\lambda\,\frac{\partial t(0,\ \tau)}{\partial x} + q_e - ct = 0, \tag{3a}$$

where the term ct takes account of all possible energy losses in the given experiment.

Equation (7) may be reduced to (1) by the substitution [2]

$$t = \exp(-c\tau)\,t(x,\ \tau), \tag{8}$$

and from (7), (8), and (1) we find

$$t(x,\ \tau) - t_0 = \frac{2q_e\exp(-c\tau)}{\lambda}\sqrt{a\tau}\ \text{ierfc}\,\frac{x}{2\sqrt{a\tau}}. \tag{9}$$

Equation (9) may be written for convenience in the form

$$t(x,\ \tau) - t_0 = \frac{2q_e\exp(-c\tau)}{\lambda}\sqrt{a\tau}\left\{\frac{\exp[-(x/2\sqrt{a\tau})^2]}{\sqrt{\pi}} - \frac{x}{2\sqrt{a\tau}}\left(1 - \text{erf}\,\frac{x}{2\sqrt{a\tau}}\right)\right\}. \tag{10}$$

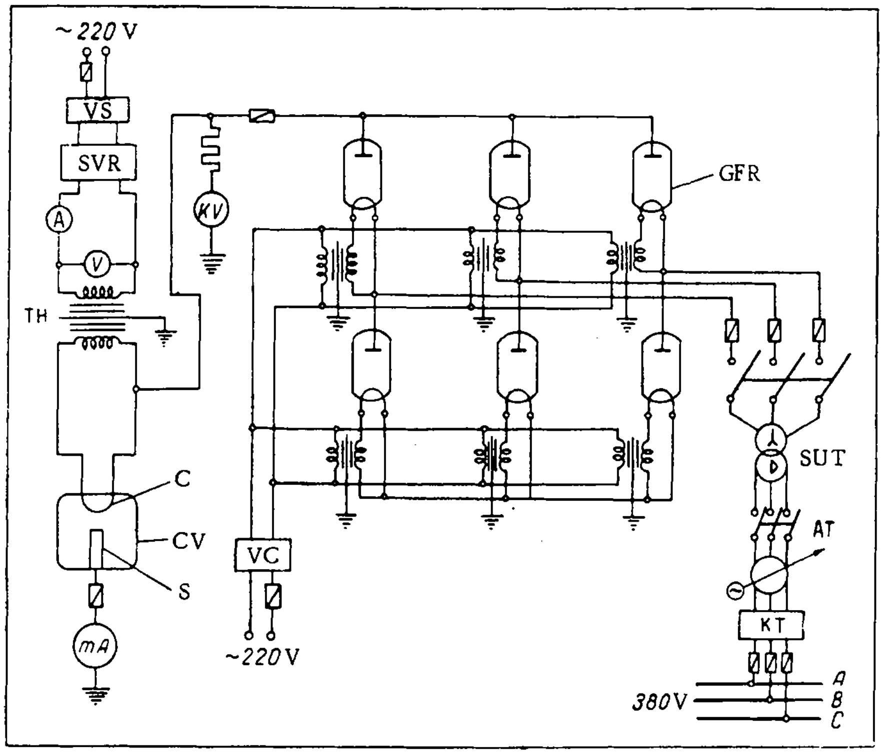
When $\frac{x}{2\sqrt{a\tau}} \leqslant 0.1$, we may neglect the term $\frac{x}{2\sqrt{a\tau}} \times \left(\text{erf}\,\frac{x}{2\sqrt{a\tau}}\right)$ with an accuracy of 1%, and we may consider the term $\frac{1}{\sqrt{\pi}}\exp\left[-\left(\frac{x}{2\sqrt{a\tau}}\right)^2\right]$ equal to $1/\sqrt{\pi}$, since the exponent is close to zero. Since the thermal diffusivities of metals are large, it is easy to choose x such that the term $x/2\sqrt{a\tau}$ will always be less than 0.1. Instead of (10) therefore, we may use the simpler expression

$$t(x,\ \tau) - t_0 = \frac{q_e\exp(-c\tau)}{\lambda}\left(\frac{2\sqrt{a\tau}}{\sqrt{\pi}} - x\right). \tag{11}$$

By measuring the dependence of temperature on time at three points on the specimen, we may determine from (11) values of diffusivity, conductivity, and the correction coefficient c at the same temperature:

$$\sqrt{a} = \frac{\sqrt{\pi}}{2}\,\frac{x_2\exp(-c\tau_2) - x_1\exp(-c\tau_1)}{\sqrt{\tau_2}\exp(-c\tau_2) - \sqrt{\tau_1}\exp(-c\tau_1)}, \tag{12}$$

$$\lambda = \frac{q_e\exp(-c\tau_1)\left(\sqrt{\tau_1/\tau_2}\,x_2 - x_1\right)}{t\left\{1 - \exp[-c(\tau_1 - \tau_2)]\sqrt{\tau_1/\tau_2}\right\}}, \tag{13}$$



Main features of the equipment for simultaneous measurement of thermal conductivity and diffusivity of metals using an electronic heater: VS—voltage stabilizer; SVR—single-phase voltage regulator; TH—heater transformer; C—cathode; VC—vacuum chamber; S—specimen; KV—kilovoltmeter; GFR—gas-filled rectifier; SUT—step-up transformer; AT—autotransformer.

$$\exp(c\tau_3)(\sqrt{\tau_1}x_2-\sqrt{\tau_2}x_1)=$$
$$=\exp(c\tau_1)(\sqrt{\tau_3}x_2-\sqrt{\tau_2}x_3)-\exp(-c\tau_2)\times$$
$$\times(\sqrt{\tau_3}x_1-\sqrt{\tau_1}x_3). \quad (14)$$

The method outlined allows the thermal conductivity and diffusivity to be measured using various means of creating a constant heat flux along the specimen (lasers, laboratory projector lamps, electronic heaters, etc.). The figure shows one possible variant using an electronic heater. The equipment works on the principle of a tube diode, in which the anode (the specimen) is heated by the flux of electrons emitted from the heated cathode. The three-phase high-voltage rectifier is hooked up to the six gas-filled rectifiers in a Larionov bridge circuit. The heater power is measured with a kilovoltmeter and milliameter. Temperature is measured by recording it at three points, with a potentiometric recorder (at low temperatures) or with a pyrometer or photographic method (at high temperatures). The specimen properties are determined by analyzing the $t=f(\tau)$ curves obtained from (14), (12), and (13).

NOTATION

q_e—heat flux density; t—temperature; x—distance to thermocouple; τ—heating time; t_0—temperature at time zero; λ—thermal conductivity of material examined; a—thermal diffusivity of material; c—coefficient describing the energy losses; x_1, x_2, x_3—distance to first, second, and third thermocouples, respectively; τ_1, τ_2, τ_3—times during which specimen is heated to temperature t at points x_1, x_2, x_3, respectively.

REFERENCES

1. A. V. Luikov, Theory of Heat Conduction [in Russian], Moscow, 1952.

2. V. N. Smirnov, A Course of Higher Mathematics, vol. 2 [in Russian], Moscow, 1953.

Polzunov Central Boiler-Turbine Institute, Leningrad.

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